Figure 4 Well Mixed Tank with Heater and Temperature Controller

An energy balance on the stirred tank system gives:

\[
\frac{dT}{dt} = \frac{WC_p (T_i - T) + q}{\rho V C_p} ; \text{ at } t = 0, \ T = T_r
\]

For this problem, we’ll ignore the facts that there could be a lag (delay) between the tank temperature (T) and the exit fluid temperature at the sensor (T_o) as well as a potential delayed sensor measured value (T_m). The output of the controller can be represented by the following equation, where \( q \) is the energy input to the tank. We’ll use a PI controller, represented by (note, for this part of the problem, let \( T = T_o = T_m \)):

\[
q = q_s + K_c (T_r - T_m) + \frac{K_i}{\tau} \int_0^t (T_r - T_m) \, dt
\]

with the steady state energy input (\( q_s \)) equal to the following:

\[
q_s = WC_p (T_r - T_is)
\]

Using the model equations and the baseline conditions given in the Table 4 below, set up this problem to solve via an ODE solver (like Polymath). Then:
a) demonstrate the open loop performance (set $Kc = 0$) of the system when the system is initially operating at design steady state temperature of 80°C, and then at $t = 1$ min, the inlet temperature ($T_i$) is suddenly changed to 55°C. Plot the temperatures to steady state. What kind of response does the system show?

b) demonstrate the closed loop performance of the system for the conditions given in a) and the baseline/control parameters form Table 4. Plot the tank temperature ($T$) vs time for a step change disturbance in the inlet temperature ($T_i$) to steady state.

c) repeat b) but find tuning parameters that roughly achieve a QAD response. An additional criteria is that the temperature in the tank doesn’t exceed 82 °C.

| Table 4  Baseline System and Control Parameters for Problem |
|-----------------|-----------------|
| $pVc_p = 4000$ kJ/°C | $Wc_p = 500$ kJ/min °C |
| $T_{is} = 60$ °C | $T_r = 80$ °C |
| $\tau_d = 1$ min | $\tau_m = 5$ min |
| $Kc = 50$ kJ/min °C | $\tau_f = 2$ min |