Pendulum Wave Machine

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Abstract

The objective of this project was to create a fun and exciting demonstration that would fascinate people of all ages and get them interested in the fields of science, technology, engineering, and mathematics (STEM). The pendulum wave machine helps demonstrate physics concepts, such as periodicity, traveling waves, standing waves, beat frequencies, and gravitational acceleration. The pendulum wave machine is composed of fifteen uncoupled pendulums with monotonically increasing string lengths. All of the pendulums have a common period of 1 minute and the frequency of oscillation of each successive pendulum increases by 1 oscillation per minute. When the pendulums are all held at a common angle and then released, each pendulum oscillates back and forth independent from one another, but when viewed as a whole, the pendulums seem to dance together in harmony at certain times, and it appears to be in complete disarray at other times. After 1 minute, this cycle will repeat itself.

Background

The idea for the pendulum waves project came from a video posted by Harvard Natural Sciences Lecture Demonstrations [1]. This video demonstrates how pendulums with a common period can be arranged to create a fun and exciting demonstration of physics concepts. The main parameters that control the period of oscillation of a pendulum are the length of the pendulum string and the angle at which the pendulum is released. By fixing the angle at which all of the pendulums are released, the string length can then be adjusted to control the period of the pendulums oscillations. The pendulum wave machine is built by selecting string lengths for each of the pendulums such that all of the pendulums have a common period of 1 minute, while each of the pendulums frequencies of oscillation vary by 1 oscillation per minute. For example, the first pendulum oscillates 60 times a minute, the second pendulum oscillates 61 times a minute, the third pendulum oscillates 62 times a minute, and so on.

Materials and Methods

The pendulum waves project was constructed using materials that can be found at your local hardware store and sporting goods store. The design consists of a 36 inch by 11.25 inch by 0.75 inch board that was used as the base of the pendulum wave machine. Four ½ inch diameter holes were drilled in the corners of the base board that would be used to support 4 -16 inch lengths of ½ inch diameter aluminum rod. The upper portion of the pendulum wave machine consists of two 11.25 inch by 3.5 inch by 0.75 inch boards that span the width of the base board and one 36 inch by 3.5 inch by 0.75 inch board that spans the length of the base board. To construct the top portion of the pendulum wave machine, the 11.25 inch by 3.5 inch by 0.75 inch boards were glued to each end of the 36 inch by 3.5 inch by 0.75 inch board, as seen in

Figure 1. The top section of the pendulum wave machine was then laid over the base boards holes, and ½ inch diameter holes were drilled in each of the corners of the 11.25 inch by 3.5 inch by 0.75 inch boards. These holes were aligned with the holes in the baseboard. Next, #216 sized steel screw eyes were placed in the top of 15 golf balls. Screw eyes were also placed on the underside of the upper portion of the pendulum wave machine, spaced at 2 inch intervals. The 15 golf balls were then suspended from these screw eyes by feeding fishing line through the screw eye on the golf ball and tying each end of the fishing line to the screw eyes that were spaced at 2 inch intervals on the upper portion of the pendulum wave machine. The vertical distance that each of the golf balls hang was carefully selected according to the string lengths listed in Table 1. This vertical distance was measured as the distance from the center of the screw eyes down to the center of the golf ball. Once the golf balls were all suspended, the wave machine was then tested by taking a 33-inch board and pulling back all of the golf balls to an angle of 45 degrees. They were then quickly released by moving the board forward and down to let the balls start swinging freely.

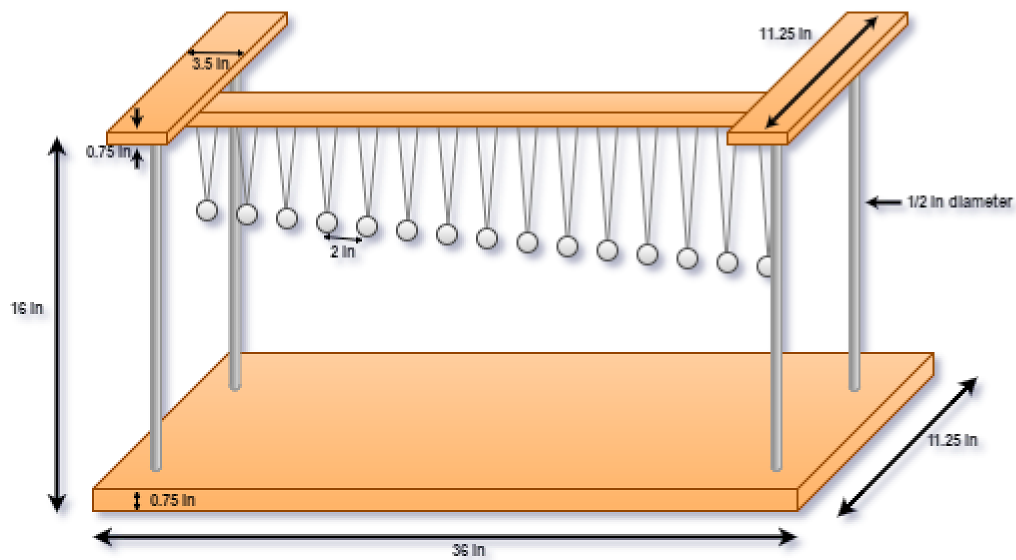

Figure 1: Pendulum Wave Machine

Results and Discussion

The period of a simple pendulum depends on the length of the pendulum, and the angle at which it is released. A very close approximation of this period can calculated using Equation 1 [2]. This equation can then be manipulated to express the length of the pendulum as a function of the period and the angle at which the pendulum is released, as seen in Equation 2. Using a gravitational acceleration of $9.81\ \frac{m}{s^2}$ and choosing a release angle of $\frac{\pi}{9}$ radians, Equation 3 is obtained. Equation 3 can then be simplified, as shown in Equation 4. This result lets us calculate the length of the pendulum for a given period.

$$T = 2\pi \sqrt{\frac{L}{g}} \left[1 + \frac{1}{16}\theta^2 + \frac{11}{3072}\theta^4\right] \qquad \textit{Equation 1}$$

$$L = g * \left[\frac{T}{2\pi\left[1 + \frac{1}{16}\theta^2 + \frac{11}{3072}\theta^4\right]}\right]^2 \qquad \textit{Equation 2}$$

$$L = 9.81 * \left[\frac{T}{2\pi\left[1 + \frac{1}{16}\left(\frac{\pi}{9}\right)^2 + \frac{11}{3072}\left(\frac{\pi}{9}\right)^4\right]}\right]^2 \ meters \qquad \textit{Equation 3}$$

$$L = 0.244722 \cdot T^2 \ meters \qquad \text{Equation 4}$$

By selecting a common period of oscillation for all of the pendulums, the pendulum waves that are created will repeat every common period. The common period of the pendulum wave machine that was created was set at 60 seconds, and the period of each individual pendulum was set according to Table 1.

Table 1: Pendulum Parameters

Pendulum Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Vertical Height (cm)	21.66	20.87	20.11	19.39	18.70	18.04	17.41	16.82	16.24	15.69	15.17	14.67	14.18	13.72	13.28
Period (s)	$\frac{60}{60}$	$\frac{60}{61}$	$\frac{60}{62}$	$\frac{60}{63}$	$\frac{60}{64}$	$\frac{60}{65}$	$\frac{60}{66}$	$\frac{60}{67}$	$\frac{60}{68}$	$\frac{60}{69}$	$\frac{60}{70}$	$\frac{60}{71}$	$\frac{60}{72}$	$\frac{60}{73}$	$\frac{60}{74}$
Frequency (oscillations per minute)	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74

Conclusions

The pendulum wave machine is a fun and exciting way to demonstrate the physics of simple pendulums and periodicity to people of all ages. It shows how combining multiple pendulums together with carefully selected periods can create a mesmerizing visual display. The pendulum wave machine is easy to operate, and it can be started by an individual person. It would make a great project for anyone who wants to help get people excited about the fields of science, technology, engineering, and mathematics (STEM).

Appendix

Matlab Simulation

```
num_samples = 100;
num_secs = 60;
time = linspace(0, 60, num_samples*num_secs);

ball(1:num_samples*num_secs,1) = sin((2*pi/(60/60)).*time);
ball(1:num_samples*num_secs,2) = sin((2*pi/(60/61)).*time);
ball(1:num_samples*num_secs,3) = sin((2*pi/(60/62)).*time);
ball(1:num_samples*num_secs,4) = sin((2*pi/(60/63)).*time);
ball(1:num_samples*num_secs,5) = sin((2*pi/(60/64)).*time);
ball(1:num_samples*num_secs,6) = sin((2*pi/(60/65)).*time);
ball(1:num_samples*num_secs,7) = sin((2*pi/(60/66)).*time);
ball(1:num_samples*num_secs,8) = sin((2*pi/(60/67)).*time);
ball(1:num_samples*num_secs,9) = sin((2*pi/(60/68)).*time);
ball(1:num_samples*num_secs,10) = sin((2*pi/(60/69)).*time);
ball(1:num_samples*num_secs,11) = sin((2*pi/(60/70)).*time);
ball(1:num_samples*num_secs,12) = sin((2*pi/(60/71)).*time);
ball(1:num_samples*num_secs,13) = sin((2*pi/(60/72)).*time);
ball(1:num_samples*num_secs,14) = sin((2*pi/(60/73)).*time);
ball(1:num_samples*num_secs,15) = sin((2*pi/(60/74)).*time);

k = 1:15;

for i = 1:num_samples*num_secs
    plot(ball(i,k),k*2,'.b','Markersize',50)
    axis([-1.5 1.5 0 32])
    title(['Time: ',num2str(round(i/100))],'fontweight','bold','fontsize',16);
    pause(0.005)
end
```

Works Cited

[1] "Pendulum Waves," Pendulum Waves, 09-Jun-2010. [Online]. Available: https://www.youtube.com/watch?v=yvkdfj9pkrq. [Accessed: 01-Jun-2016].

[2] "Large Amplitude Pendulum," Large Amplitude Pendulum. [Online]. Available: http://hyperphysics.phy-astr.gsu.edu/hbase/pendl.html. [Accessed: 01-Jun-2016].